

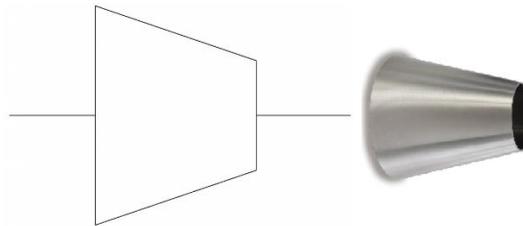
Modelling truncated cones 2D design by means Rstudio and 3D assembly of

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Abstract

Building a 3D straight cone requires the help of a compass, a protractor, and a surface to draw it. This approach is one possible way, but there are others. Subsequently, cutting out this outline and folding it allows us to arrive at a 3D object. Constructing a truncated right cone is less trivial than the preceding but equally achievable. For a school level, the above is sufficient. However, at the technological level, the design of parts and pieces of propulsion vessels, boiler-making parts, or pipelines reaches a higher complexity. A common problem in the latter three is constructing truncated straight cones with certain geometrical boundary conditions. They are called concentric reduction, as seen in the following figure.



In this paper, an alternative way of 2D design and 3D assembly is examined, applicable to a family of straight cone trunks and, in the process, modeling through Rstudio the different geometrical conditions to achieve an optimal fit of parts. The other available approaches are briefly discussed so that the reader can decide which strategy is functionally closer, according to the reader's experience, to the construction of a concentric reduction.

1 Introduction

A problem that tends to arise in the design of structural modules for aerospace vessels and also in plumbing, woodworking, industrial ventilation, plumbing, piping, and machining is how to correctly assemble truncated cone geometric components so that the sections fit together correctly, according to the drawn plan.

On the other hand, thin-walled conical structures are widely used in aerospace, offshore, civil and other engineering fields. Parts of space launcher transport systems are one example for the application of conical shells [1, 2]. Nowadays, the analysts still use empirically based lower-bound methods such as the NASA [3] SP-8007 for cylinders and SP-8019 for cones [4] to calculate the required knock-down factors (KDFs), which do not include important mechanical properties of laminated composite materials [5], such as the stacking sequence.

Generally, in the plans of these modules, it is known what are the diameters of both truncated sections, the height of the truncated cone, and the dimension of the generatrix, but what is the geometric condition in 2D that ensures it meets exactly the 3D requirements, following the proposed plan for the section?

In this paper, using Rstudio, a method is proposed to model and fully ensure the 2D geometric boundary conditions, using the generatrix of the truncated cone, the arc sections of each truncated circle, S and S' , the angle θ and from there, to achieve a 3D assembly of the sections that fit correctly, following the 2D layout of the plane.

2. Materials and methods

For the 2D drawing, it a square is required, a carpenter's compass, see Figure 1, and a protractor. Modeling in Rstudio requires a 36-bit PC and a screen with good graphic resolution.



Figure 1. Carpenter Compass

3. Results and Discussion

Case 1. Right Cone

Let us start with the simplest Figure. A right cone is essentially constituted by the radius r and the generatrix g ; see Figure 1. The right cone is represented by the generatrix g that runs from the point of the circumferential section to the apex of the right cone, the height h that describes the distance from the center of the circumferential section to the apex of the cone, the radius r of the circumferential section and d the diameter of the circumferential section. In other words, a right cone is a solid of revolution generated by the rotation of a right triangle around one of its legs. The circle formed by the other leg is called the base, and the point where the generatrices meet is called the vertex.

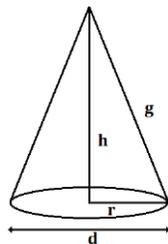


Figure 2. Right Cone

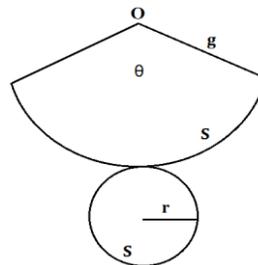


Figure 3. Right cone plane of Figure 2

How is a 2D drawing made from Figure 2 for the case of the straight cone? Usually, the generator g , the diameter of the cone, is measured; then, with the help of a compass of aperture equal to g , it is swept, starting from the origin O , joining the right ray with the left ray, following an arc element of size S , whose angular distance θ in radians, allows obtaining the magnitude:

$$S = g \cdot \theta \quad /1/$$

Specifically, this same arc S measures precisely the perimeter $S = 2\pi r$ of the circular section, with $r = d/2$, half the diameter, see Figure 2. The demonstration is left for the truncated cone section.

Case 2. Truncated Cone

This situation is similar to Figure 2, except the apex has been cut off parallel to the circular section base. A new generatrix appears at a new height and another circular section of a smaller radius than the first. See Figures 4 and 5. The generatrix is measured with a graduated ruler. A truncated cone can also be described as a solid of revolution generated by rotating a rectangular trapezium taking as its axis of rotation its side perpendicular to the bases. Figure 5 follows that $S = 2\pi r$ and $S' = 2\pi r'$ and $S' > S$ given that $r' > r$ and r' are determined directly from the 2D plane or by their diameters. In the case of a pipe, its diameters are measured employing a foot meter.

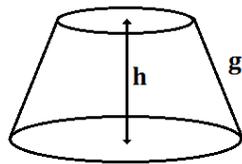


Figure 4. Truncated Cone

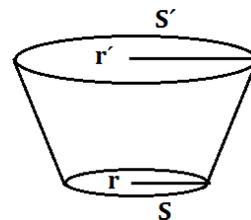


Figure 5. Inverted Truncated Cone

Solving this new situation in a 2D plane involves a conceptual map proposed in Figure 6. Here it is convenient to establish two new lengths, minor radius $r'_<$ and major radius $r'_>$. How to compute $r'_<$ and $r'_>$ from S and S' following the 2D plane? This issue will be discussed below.

As shown in this Figure, the product of the minor radius $r'_<$ by the angle θ generates the arc element S . That is $S = r'_< \cdot \theta$. The same for. The boundary condition is that :

$$r'_< \cdot \theta = 2\pi r \quad \text{para } S \quad /2/$$

and

$$r_{>} \cdot \theta = 2\pi r' \text{ para } S' \quad /3/$$

Therefore, this means that the arc elements between Figure 5 and 6 are equal.

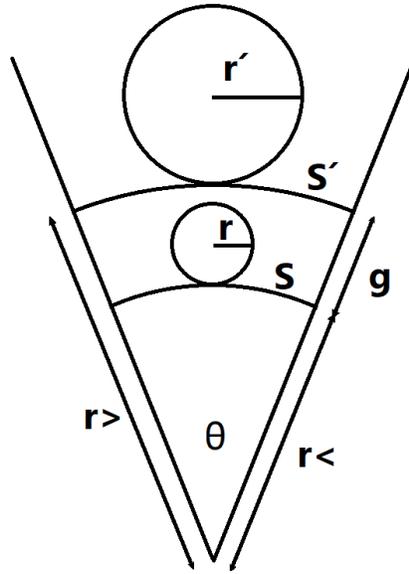


Figure 6. 2D projection of the truncated cone

Furthermore, the 2D plane specifies the value of S and S' . That is S , S' and g , is known in advance, as is r and r' . The problem is to know θ since θ is common to both. In additional attention to Figure 5, it is possible to deduce that for any angle θ , there are infinitely many solutions of larger and smaller radii for the truncated cone, but one and only one that satisfies eq:

$$\frac{S'}{r_{>}} = \frac{S}{r_{<}} = \theta \quad \text{for } S, S' \text{ and } g \text{ are known} \quad /4/$$

However $r_{>} = r_{<} + g$, then the above equation leaves us:

$$\frac{S'}{(r_{<} + g)} = \frac{S}{r_{<}} \quad /5/$$

Leading to:

$$r_{<} \cdot S' = S(r_{<} + g) \quad /6/$$

where clearing $r_{<}$, it is obtained:

$$r_{<} = \left(\frac{S}{S' - S} \right) g, \text{ exact solution } /7/$$

and following with $r_{>} = (r_{<} + g)$, plus a proper factorization, it is possible to obtain

$$r_{>} = \left(\frac{S'}{S' - S} \right) g \quad /8/$$

Afterward, it is necessary to calculate the required angle (in radians) according to:

$$\theta = \frac{S}{r_{<}} \quad /9/$$

When applying this to Figure 6, fixing the origin at O, tracing two rays that exactly inscribe the calculated angle. Afterward, plot with a measuring longitude compass $r_{<}$ and sweep the angle from one ray to reach the other one. With that, the length of S is fixed. Widening the aperture of the compass to the sum of $r_{<} + g$ and placing one end at the origin O, moving an arc segment from one ray to intersect the other. With that, the length of S' and the exact truncated cone are fixed, with its mantle and circular sections according to the 2D layout. The figure is cut out and assembled in 3D. With this, the construction of the geometrical part is completed in compliance with the drawing.

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Particular case: demonstration of the formula for the simple straight cone

Taking the second equation:

$$r_{>} = \left(\frac{S'}{S' - S} \right) g$$

Given that in the case of the simple cone, there is only one partition; it is assumed that $S = 0$ and considering only S' .

In that case:

$$r_{>} = g$$

Thus, the initial conditions of equation /1/ and Figure 2 are recovered.

In the context of plumbing, boiler making, gasfitting, and pipeline, this issue is problematic and has been extensively and exhaustively solved, using a ruler and compass, in an empirical format. The current proposal is different, but it is still an open problem.

Using a 3D printer coupled with AUTOCAD® represents another way to design and construct parts such as those shown in this work [6-8]. SPACEX© could be the example, as one of the most innovative companies in aerospace material. However, new design approaches are required to take full advantage of composite materials [9, 10]. The author thinks that this contribution, although modest, considering that this is a problem sufficiently addressed in pipeline engineering, gives a greater diversity of solutions for the same problem, which could be a possible strength for engineering.

3. R code

```
# Truncated cone design, angle and radius

install.packages('ggplot2') # Install libraries
library(ggplot2)           # load library

# Input data

g = 0 # Length of generatrix
g = 7.9

d1 = 0 # Diameter smaller section
r1 = 0
S1 = 0
d1 = 4.5 # radius smaller section
r1 = d1/2
S1 = 2*pi*(d1/2)# First perimeter

d2 = 0 # Diameter bigger section
r2 = 0
S2 = 0
d2 = 7.6 # radius bigger section
r2 = d2/2
S2 = 2*pi*(d2/2) # Second perimeter

rminor = 0 # Calculus of r minor
rminor = (S1/(S2-S1))*g

rmajor = 0 # Calculus of r major
rmajor = (S2/(S2-S1))*g

theta = 0 # Calculus of theta
theta = S1/rminor # in radians
thetadegree = 0 # Convert to degree
thetadegree = theta*180/pi

print(paste("r1 =", r1))
print(paste("r2 =", r2))
print(paste("S1 =", S1))
print(paste("S2 =", S2))
print(paste("theta =", theta))
print(paste("thetadegree =", thetadegree))
```

4. Concluding remarks

Assisted by the 2D drawing of the project of parts and pieces that require knowing the exact dimensions of the truncated cone under study and knowing the diameters of the circular sections that are needed, the generatrix of the truncated cone and the assistance of Rstudio through the equations:

$$r_{<} = \left(\frac{S}{S' - S} \right) g \quad r_{>} = \left(\frac{S'}{S' - S} \right) g \quad \text{and the angle } \theta \text{ given by: } \theta = \frac{S}{r_{<}}$$

Figure 6 sine qua non condition makes it possible to model the truncated cone according to the layout of the initial planes.

Acknowledgement

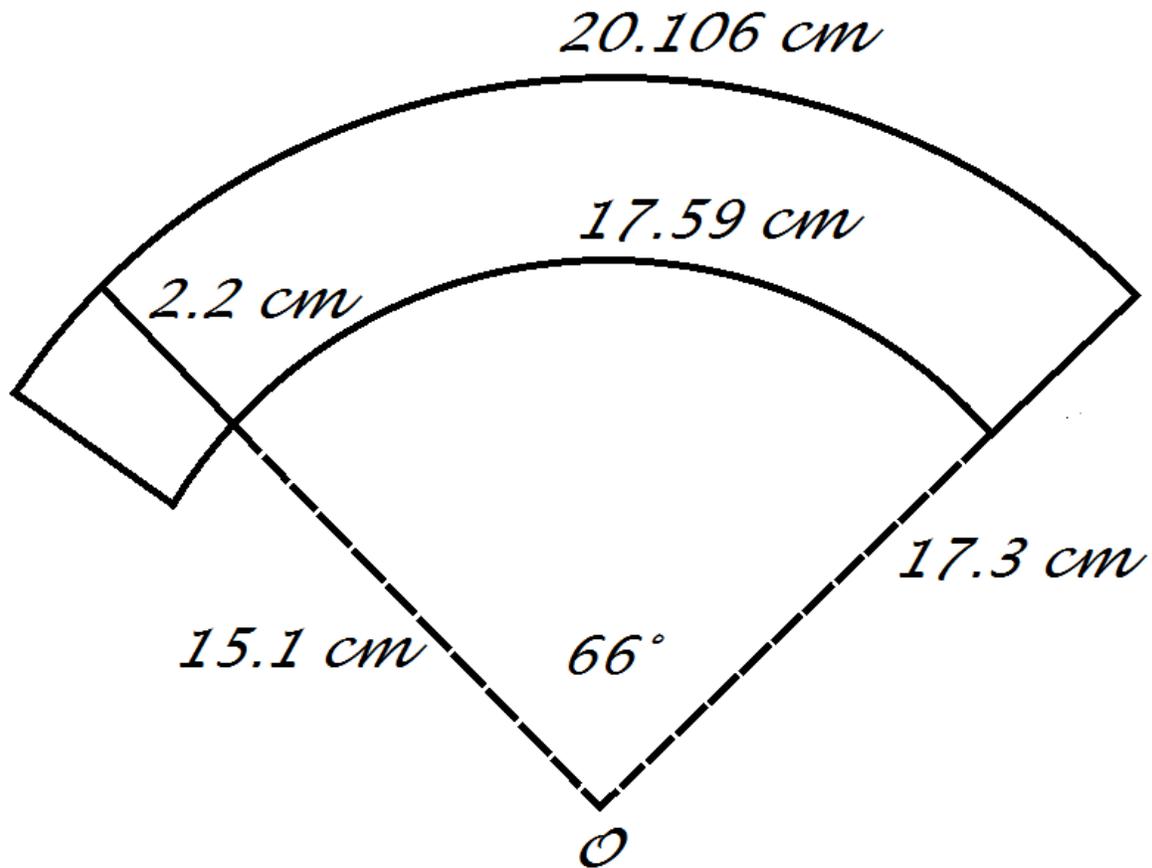
*The author, CGR, acknowledges **Laboratorio de Computación Cuántica** for dedicating time and lending the facilities during the development of the present work.*

ANEXO 1.

Example

Set this data in R code and run, you should obtain this one:

$$\begin{array}{llll} g = 2.2 \text{ cm} & \theta = 66^\circ & & \\ d' = 6.40 \text{ cm} & r' = 3.2 \text{ cm} & S' = 20.106 \text{ cm} & r_s = 17.3 \text{ cm} \\ d = 5.60 \text{ cm} & r = 2.8 \text{ cm} & S = 17.59 \text{ cm} & r_c = 15.1 \text{ cm} \end{array}$$



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